

Property Tax Levy Time Series Model

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Overview

This code fits a variation of a classical ARIMA time series model to the East Greenwich property tax levy for the years 1992-2025.

It uses a hierarchical (partial pooling) Bayesian model with the following structure:

Given a vector of observed property tax values (y_1, y_2, \dots, y_n) , the model is:

$$y_{i+1} = y_i + \beta_j + e_i \quad i = 2, 3, \dots, n, \quad j = 1, 2, 3$$

Where:

- y_i is the property tax levy in year i
- β_1 is the expected dollar increment in normal years from 2004 to 2025
- β_2 is the expected dollar increment in years 2018-2020 and 2023 (level-fund or near level-fund)
- β_3 is the expected dollar increment in the years 1992 to 2003
- e_i is the random error or noise term in year i assumed to have expected value zero and standard deviation σ_e .

R - Get the tax levy data

```
library(ggplot2)
library(rstan)
```

```

## Loading required package: StanHeaders
##
## rstan version 2.32.5 (Stan version 2.32.2)
## For execution on a local, multicore CPU with excess RAM we recommend
calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend
calling
## rstan_options(auto_write = TRUE)
## For within-chain threading using 'reduce_sum()' or 'map_rect()' Stan
functions,
## change 'threads_per_chain' option:
## rstan_options(threads_per_chain = 1)

library(shinystan)

## Loading required package: shiny
##
## This is shinystan version 2.6.0

library(rlist)

ll = list.load('Levy_additive_AR1.RDS')

ls = ll                                     #retrieve data extract results
ptaxdf = ls$ptaxdf                         #get dataframe

levy = c(ptaxdf$levy,64.747)               #add 2025 numbers to dataframe
year = c(ptaxdf$year,2025)
aix1 = c(ptaxdf$aix1,1)
aix2 = c(ptaxdf$aix2,1)
ptaxdf2=data.frame(year,levy,aix1,aix2)   #new dataframe with 2025 added

```

Call STAN

Call STAN to run the model

```

last_data_year = 2025                       #last data year
forecast_to    = 2037                       #last year of forecast

Ndata = last_data_year -1992 + 1            #number of data points
years_ahead = forecast_to - last_data_year  #years to forecast
aix = ptaxdf2$aix2                          #set up STAN parameters
aix = c(aix,rep(aix[length(aix)],Ndata+years_ahead-length(aix)))
y = ptaxdf2$levy[1:Ndata]

```

```

Nyears = min(years_ahead + Ndata,1000)
aixin = aix[1:Nyears]
n_params = length(table(aixin[1:Nyears]))

chains <- 4

#call STAN
gp <- stan("Levy_time_series.stan",
           data=list(N=Ndata,y=y,Na=years_ahead,n_params=n_params,aix=aixin),
           control=list(max_treedepth=18),
           chains=chains,cores=4,iter=8000)

#launch shinystan(gp)
#summary(gp) see Appendix 1
pd = rstan::extract(gp)

```

Parameter Quantiles, Fitted Values, and Residuals

```

#proposed 2025 levy 64.747
mean(pd$yp[,34]) #expected 2025 levy

## [1] 64.08815

quantile(pd$yp[,34],c(.25,.5,.75,.9,.95,.99)) #quartiles, 90th, 95th, 99th percentiles for 2025

##      25%      50%      75%      90%      95%      99%
## 63.68295 64.08827 64.49179 64.85237 65.07399 65.51462

sum(pd$yp[,34]<=64.747)/length(pd$yp[,34]) #percentile for 2025 proposed levy

## [1] 0.8634375

for (i in 1:length(pd$beta[1,])){
  print(i)
  print(quantile(pd$beta[,i],c(.25,.5,.75)))
  print(mean(pd$beta[,i]))
  print(sd(pd$beta[,i]))
}

## [1] 1
##      25%      50%      75%
## 1.745496 1.835814 1.927368
## [1] 1.835147
## [1] 0.1375572

```

```

## [1] 2
##          25%          50%          75%
## -0.14790316  0.04182023  0.23319182
## [1] 0.04733724
## [1] 0.2939665
## [1] 3
##          25%          50%          75%
## 1.076306 1.190890 1.306217
## [1] 1.189738
## [1] 0.1732965

mean(pd$sigma_e)

## [1] 0.5824415

sd(pd$sigma_e)

## [1] 0.07950644

pstart = Ndata+1
pend = Nyears

predicted = c()
for (i in pstart:pend){
  predicted = c(predicted,mean(pd$yp[,i]))
}
predicted = round(predicted,1)
fitted = c()
for (i in 1:Ndata){
  fitted = c(fitted,mean(pd$yp[,i]))
}
ptaxdf2$fitted = fitted
residuals = fitted - ptaxdf2$levy
ptaxdf2$residuals = residuals
mse = sum(residuals^2)/length(residuals)

```

Plot Actual and Fitted

```

ptaxdf2
##   year   levy aix1 aix2  fitted  residuals
## 1  1992 18.12136   1   3 18.11267 -0.008693746
## 2  1993 18.86825   1   3 19.30213  0.433884424
## 3  1994 20.15463   1   3 20.05903 -0.095598947

```

```

## 4 1995 21.60600 1 3 21.35043 -0.255569512
## 5 1996 22.68903 1 3 22.79779 0.108765093
## 6 1997 23.12129 1 3 23.86973 0.748438279
## 7 1998 24.16347 1 3 24.31034 0.146875728
## 8 1999 25.55655 1 3 25.35311 -0.203440166
## 9 2000 26.92341 1 3 26.74901 -0.174404828
## 10 2001 29.40625 1 3 28.11722 -1.289033298
## 11 2002 30.63086 1 3 30.59626 -0.034603431
## 12 2003 31.40400 1 3 31.82094 0.416940839
## 13 2004 33.56387 1 1 33.23469 -0.329180619
## 14 2005 35.54766 1 1 35.39749 -0.150172037
## 15 2006 38.04237 1 1 37.38497 -0.657401941
## 16 2007 38.86538 1 1 39.87442 1.009034348
## 17 2008 41.52052 1 1 40.69373 -0.826789531
## 18 2009 43.62369 1 1 43.35111 -0.272579216
## 19 2010 45.58809 1 1 45.46392 -0.124169419
## 20 2011 47.20572 1 1 47.42558 0.219851072
## 21 2012 48.08608 1 1 49.04251 0.956422021
## 22 2013 49.67504 1 1 49.91875 0.243700683
## 23 2014 51.84579 1 1 51.51181 -0.333982710
## 24 2015 53.34561 1 1 53.68292 0.337313208
## 25 2016 55.44380 1 1 55.18717 -0.256632310
## 26 2017 57.01941 1 1 57.27571 0.256305588
## 27 2018 56.16991 2 2 57.06822 0.898312673
## 28 2019 55.98191 2 2 56.21646 0.234550050
## 29 2020 56.88020 2 2 56.03070 -0.849492187
## 30 2021 57.93378 1 1 58.71149 0.777714363
## 31 2022 59.94980 1 1 59.77598 -0.173816929
## 32 2023 59.88519 2 2 60.00040 0.115207410
## 33 2024 62.25693 1 1 61.72056 -0.536367545
## 34 2025 64.74700 1 1 64.08815 -0.658852953

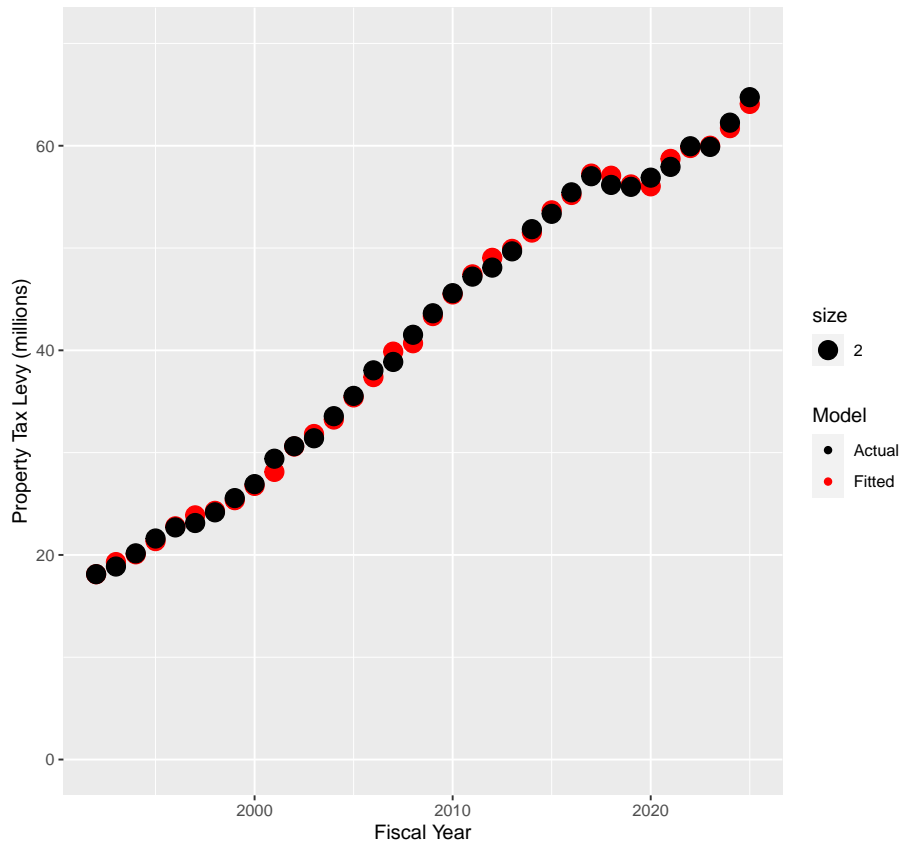
yr = rep(ptaxdf2$year,2)
grp=c(rep('fitted',nrow(ptaxdf2)),rep('actual',nrow(ptaxdf2)))
levy = c(ptaxdf2$fitted,ptaxdf2$levy)

gdf = data.frame(yr,grp,levy)

ggplot(gdf, aes(x=yr, y=levy)) +
  geom_point(aes(color=grp,size=2)) + ylim(0,70) +
  scale_color_manual(name='Model', labels=c('Actual', 'Fitted'),
                    values=c('black', 'red')) +
  labs(x = "Fiscal Year", y="Property Tax Levy (millions)",
       title = "Property Tax Levy - A Third of a Century of History (1992-2025)",
       subtitle = "Actual (black) and Fitted (red) Values")

```

Property Tax Levy – A Third of a Century of History (1992–2025)
Actual (black) and Fitted (red) Values



```
fname = paste("images/Property_Tax_History_Through_2025_actual_and_fitted_", Sys.Date(), ".png")
fname

## [1] "images/Property_Tax_History_Through_2025_actual_and_fitted_2024-05-06.png"

ggsave(fname, width = 8, height = 5, units = "in")
```

Forecast

Run forecast

```
yrsp = length(pd$yp[1,]) - nrow(ptaxdf2) #years ahead
last_data_year = ptaxdf2$year[nrow(ptaxdf2)]
last_data_year + yrsp
```

```

## [1] 2037

yr = seq(last_data_year+1,last_data_year+yrsp) #forecast years
grp = rep('Forecast',yrsp)
levy = c()
lv01 = c()
lv025 = c()
lv05 = c()
lv25 = c()
lv75 = c()
lv95 = c()
lv975 = c()
lv99 = c()
for (i in (1+nrow(ptaxdf2)):(nrow(ptaxdf2)+yrsp)){
  levy = c(levy,mean(pd$yp[,i]))
  lv01 = c(lv01,quantile(pd$yp[,i],0.01))
  lv025 = c(lv025,quantile(pd$yp[,i],0.025))
  lv05 = c(lv05,quantile(pd$yp[,i],0.05))
  lv25 = c(lv25,quantile(pd$yp[,i],0.25))
  lv75 = c(lv75,quantile(pd$yp[,i],0.75))
  lv95 = c(lv95,quantile(pd$yp[,i],0.95))
  lv975 = c(lv975,quantile(pd$yp[,i],0.975))
  lv99 = c(lv99,quantile(pd$yp[,i],0.99))
}

gdff = data.frame(yr,grp,levy)
gdfpred = data.frame(yr,levy,lv01,lv025,lv05,lv25,lv75,lv95,lv975,lv99)
gdffpred

##      yr      levy      lv01      lv025      lv05      lv25      lv75      lv95      lv975
## 1  2026 66.58041 65.12968 65.38565 65.58770 66.17515 66.98472 67.57763 67.77964
## 2  2027 68.41849 66.33344 66.67048 66.97875 67.83906 69.00616 69.85098 70.16652
## 3  2028 70.25623 67.60716 68.05733 68.43560 69.53701 70.98063 72.05493 72.41526
## 4  2029 72.09122 68.96046 69.50369 69.96760 71.22405 72.94813 74.20384 74.61900
## 5  2030 73.93298 70.37261 70.99139 71.49441 72.94488 74.92003 76.37059 76.88148
## 6  2031 75.76659 71.72539 72.46032 73.03299 74.67650 76.87332 78.46436 79.00914
## 7  2032 77.60152 73.19930 73.92794 74.58335 76.40043 78.82332 80.58675 81.24093
## 8  2033 79.43937 74.62075 75.45544 76.15181 78.11669 80.77106 82.70092 83.38291
## 9  2034 81.26909 76.09321 76.99914 77.69632 79.83659 82.69442 84.81437 85.50416
## 10 2035 83.10220 77.60448 78.46145 79.27386 81.57472 84.64489 86.88587 87.69003
## 11 2036 84.93428 78.92472 80.02887 80.81493 83.32681 86.58287 88.98388 89.79609
## 12 2037 86.77044 80.34482 81.49528 82.38829 85.04056 88.51468 91.08914 91.91651
##      lv99
## 1  68.03734
## 2  70.50738
## 3  72.84167

```

```

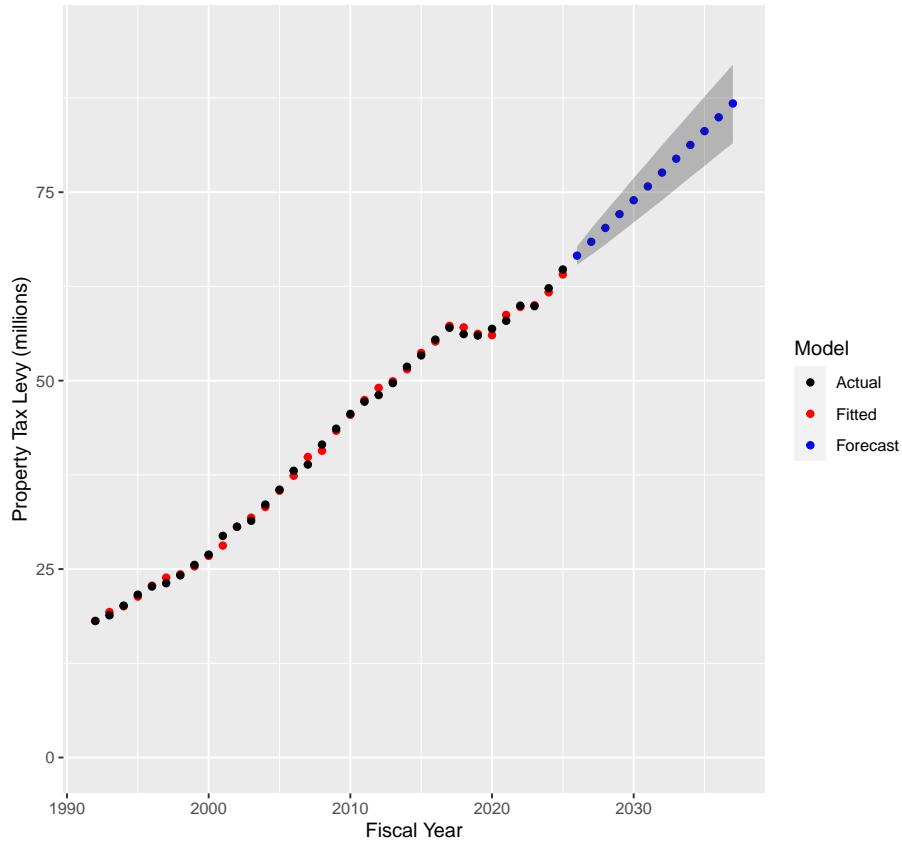
## 4 75.16210
## 5 77.46741
## 6 79.67780
## 7 81.88874
## 8 84.14486
## 9 86.28649
## 10 88.47079
## 11 90.72734
## 12 92.94869

gdf2 = rbind(gdf,gdff)
p = ggplot(gdf2, aes(x=yr, y=levy)) +
  geom_point(aes(color=grp)) + ylim(0,95) +
  scale_color_manual(name='Model', labels=c('Actual', 'Fitted', 'Forecast'),
                    values=c('black', 'red', 'blue')) +
  labs(x = "Fiscal Year", y="Property Tax Levy (millions)",
       title = "Property Tax Levy Status Quo Forecast 2026-2037 with 95% Credible Intervals",
       subtitle = "Actual (black), Fitted (red), and Status Quo Forecast (blue) Values")

p + geom_ribbon(data=gdfpred, aes(ymin=lv025,ymax=lv975), alpha=0.3)

```

Property Tax Levy Status Quo Forecast 2026–2037 with 95% Credible Intervals
 Actual (black), Fitted (red), and Status Quo Forecast (blue) Values



```
fname = paste("images/Property_Tax_History_Through_2025_13_year_status_quo_forecast_", Sys.Date())
fname
## [1] "images/Property_Tax_History_Through_2025_13_year_status_quo_forecast_2024-05-06.png"
ggsave(fname, width = 8, height = 5, units = "in")
```

Prediction Barcharts

Barcharts

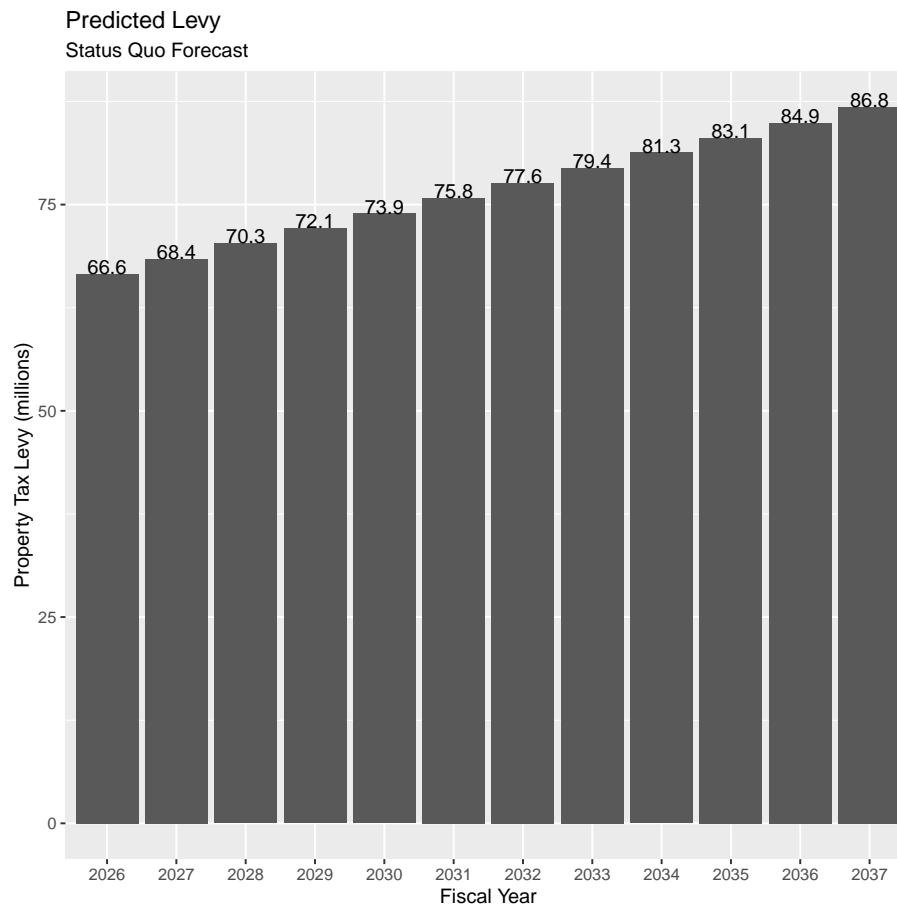
```
pyears = as.factor(seq(2026, 2037))
deltas = 1.8 * (1 + length(pyears) - seq(1: length(pyears)))
```

```

bcdf = data.frame(pyears,predicted,deltas)

ggplot(bcdf, aes(x = pyears, y = predicted)) + # Plot with values on top
  geom_bar(stat = "identity") +
  geom_text(aes(label = round(predicted,1)), vjust = 0) +
  labs(x = "Fiscal Year", y="Property Tax Levy (millions)",
       title="Predicted Levy",
       subtitle="Status Quo Forecast")

```



```

fname = paste("images/Property_Tax_History_Through_2025_13_year_status_quo_forecast_barchart",
fname

## [1] "images/Property_Tax_History_Through_2025_13_year_status_quo_forecast_barchart_2024-0

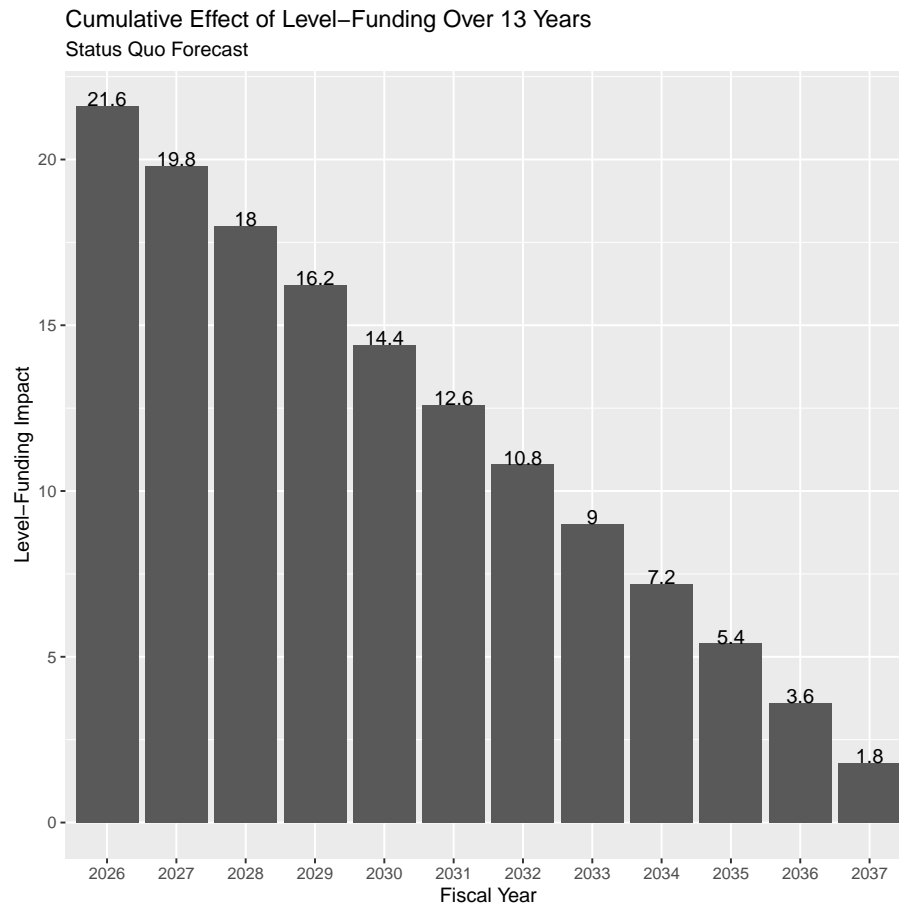
ggsave(fname, width = 8, height = 5, units = "in")

```

```

ggplot(bcdf, aes(x = pyears, y = deltas)) + # Plot with values on top
  geom_bar(stat = "identity") +
  geom_text(aes(label = round(deltas,1)), vjust = 0) +
  labs(x = "Fiscal Year", y="Level-Funding Impact",
       title="Cumulative Effect of Level-Funding Over 13 Years",
       subtitle="Status Quo Forecast")

```



```

fname = paste("images/Property_Tax_History_Through_2025_13_year_status_quo_level_fund_impact",
fname

## [1] "images/Property_Tax_History_Through_2025_13_year_status_quo_level_fund_impact_2024-0

ggsave(fname, width = 8, height = 5, units = "in")

```

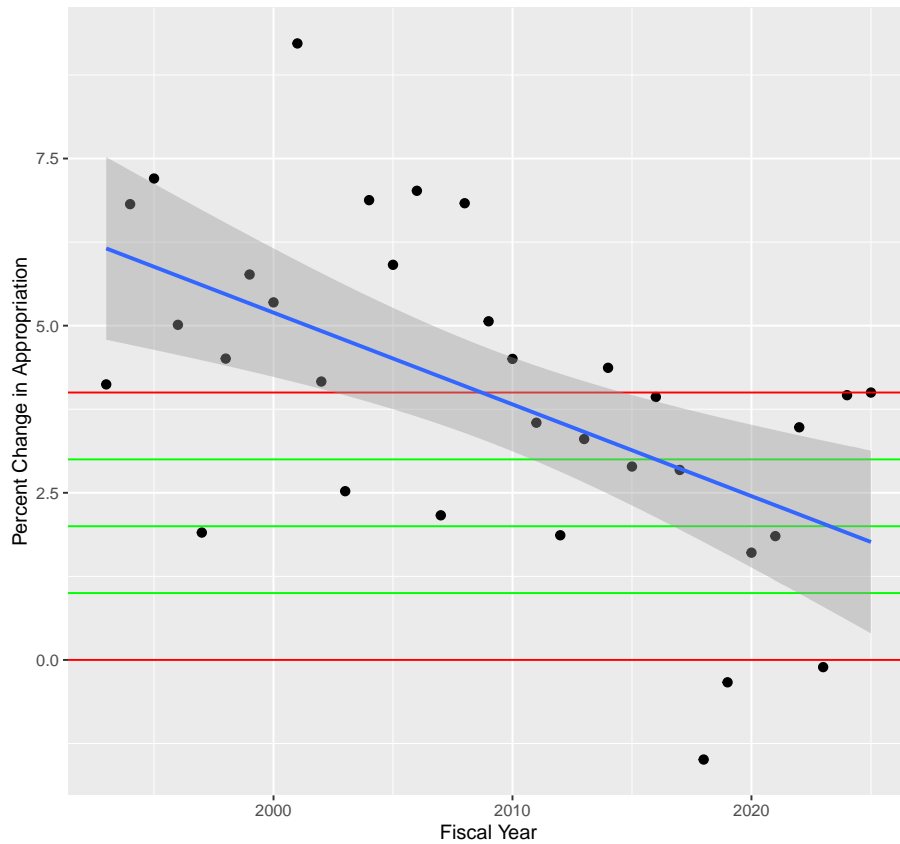
Linear Characteristics

Plot yeat-to-year percent changes and deltas

```
pctchg = 100*(ptaxdf2$levy[2:nrow(ptaxdf2)]/ptaxdf2$levy[1:(nrow(ptaxdf2)-1)]-1) #year over
pyr = ptaxdf2$year[2:nrow(ptaxdf2)] #year vector
pcdf = data.frame(pyr,pctchg) #data frame for ggplot2
ggplot(pcdf, aes(x=pyr, y=pctchg)) +
  geom_point(size=2) + geom_hline(yintercept=3,size=0.5,color='green') +
  geom_hline(yintercept=4, size=0.5, color='red') +
  geom_hline(yintercept=2, size=0.5, color='green') +
  geom_hline(yintercept=1, size=0.5, color='green') +
  geom_hline(yintercept=0, size=0.5, color='red') +
  geom_smooth(method='lm', formula= y~x) +
  labs(x = "Fiscal Year", y="Percent Change in Appropriation",
       title = "Year over Year Percent Change for School Appropriation",
       subtitle="1998-2024 Red: 0%, 4% Green: 1%, 2%, 3% Blue: Regression Line")

## Warning: Using 'size' aesthetic for lines was deprecated in ggplot2
## 3.4.0.
## i Please use 'linewidth' instead.
## This warning is displayed once every 8 hours.
## Call 'lifecycle::last_lifecycle_warnings()' to see where this warning
## was
## generated.
```

Year over Year Percent Change for School Appropriation
 1998–2024 Red: 0%, 4% Green: 1%, 2%, 3% Blue: Regression Line



```

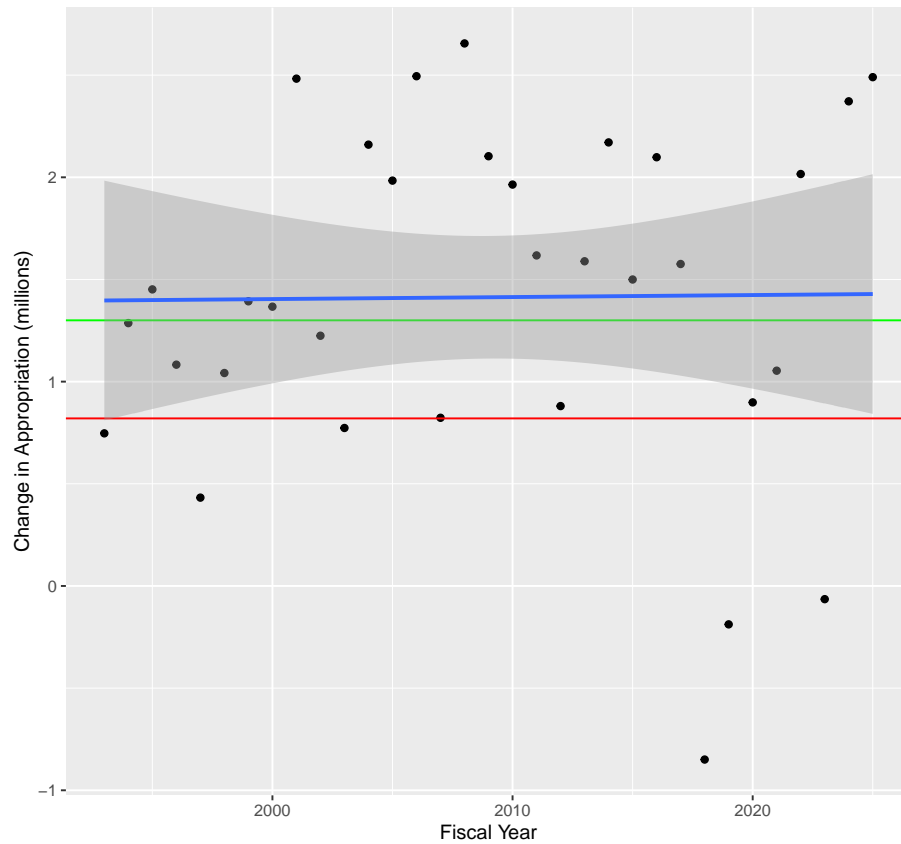
fname = paste("images/School_Appropriation_History_Year_over_Year_Percent_Change_", Sys.Date)
fname

## [1] "images/School_Appropriation_History_Year_over_Year_Percent_Change_2024-05-06.png"
ggsave(fname, width=8, height=5, units="in")

delta = ptaxdf2$levy[2:nrow(ptaxdf2)] - ptaxdf2$levy[1:(nrow(ptaxdf2)-1)] #year to y
dyr = ptaxdf2$year[2:nrow(ptaxdf2)] #year vector
ddf = data.frame(dyr, delta) #data frame for ggplot2
ggplot(ddf, aes(x=dyr, y=delta)) +
  geom_point() + geom_hline(yintercept=1.3, size=0.5, color='green') +
  geom_hline(yintercept=0.82, size=0.5, color='red') +
  geom_smooth(method='lm', formula= y~x) +
  labs(x = "Fiscal Year", y="Change in Appropriation (millions)",
       title = "School Appropriation Year to Year Deltas 1998-2024",
       subtitle = "Green: $1.3m Red: $820k Blue: Regression Line")
    
```

School Appropriation Year to Year Deltas 1998–2024

Green: \$1.3m Red: \$820k Blue: Regression Line



```
fname = paste("images/School_Appropriation_History_Year_to_Year_Delta_", Sys.Date(), ".png", sep="")
save_png(fname)

## [1] "images/School_Appropriation_History_Year_to_Year_Delta_2024-05-06.png"

ggsave(fname, width=8, height=5, units="in")
```

Save results for future reference

```
savelist = list(ptaxdf=ptaxdf2, gp=gp, mse=mse)
saveRDS(savelist, paste("Levy_time_series_", Sys.Date(), ".RDS", sep=""))

## [1] "Levy_time_series_2024-05-06.RDS"
```

```
list.save(savelist,fname)
```

STAN Model File

This is the STAN model file:

```
get_stanmodel(gp)

## S4 class stanmodel 'anon_model' coded as follows:
## // AR(1) model for property tax levy
## data {
##   int<lower=2> N;                // observed data points
##   int<lower=1> Na;              // data points ahead
##   vector[N] y;                 //levy
##   int<lower=1> n_params;        //number of alpha parameters
##   int aix[N+Na];               //alpha parameter index
## }
## parameters {
##   real a;                       //hyperparameter for beta
##   real<lower=0> b;              //hyperparameter for sigma_beta
##   vector[n_params] beta;
##   vector<lower=0>[n_params] sigma_beta; //vector of sigma_beta parameters
##   real<lower=0> sigma_e;        //residual standard error
## }
## model {
##   a ~ normal(0,1);              //normal prior for hyperparameter a
##   b ~ normal(0,1);              //normal prior for hyperparameter b
##   beta ~ normal(a,b);           //normal prior for betas
##   sigma_beta ~ normal(0,1);     //half-normal prior for sigma_betas
##   sigma_e ~ normal(0,1);        //half-normal prior for sigma_e
##   for (i in 2:N){
##     y[i] ~ normal(y[i-1] + beta[aix[i]],sigma_e); //model for y
##   }
## }
## generated quantities{
##   vector[N+Na] yp;              //fitted and posterior predicted
##   print(N);                     //number of observed data points
##   print(Na);                    //number of years ahead to predic t
##   yp[1] = normal_rng(y[1],sigma_e); //initial value for autoregressive
##   for (i in 2:N+1){
##     yp[i] = normal_rng(y[i-1] + beta[aix[i]],sigma_e); //fitted
##   }
##   for (i in (N+2):(N+Na)){
##     yp[i] = normal_rng(yp[i-1] + beta[aix[i]],sigma_e); //predicted
```

```
## }  
## }
```

Appendix 1: Posterior Draw Summary

```
summary(gp)  
  
## $summary  
##           mean      se_mean      sd      2.5%      25%  
## a           0.75835750 0.0053525131 0.54070556 -0.44403848 0.4406816  
## b           1.03655913 0.0041577184 0.42849517 0.44138898 0.7241443  
## beta[1]      1.83514717 0.0012001093 0.13755719 1.56310088 1.7454961  
## beta[2]      0.04733724 0.0027489629 0.29396646 -0.52357981 -0.1479032  
## beta[3]      1.18973765 0.0014580741 0.17329652 0.84136862 1.0763062  
## sigma_beta[1] 0.79348134 0.0052919662 0.61195423 0.02471417 0.3027102  
## sigma_beta[2] 0.79783206 0.0048876015 0.60834890 0.03165298 0.3139772  
## sigma_beta[3] 0.79609355 0.0049083919 0.59941910 0.03143523 0.3248923  
## sigma_e      0.58244149 0.0007347809 0.07950644 0.45010424 0.5262583  
## yp[1]       18.11266925 0.0046313538 0.58713250 16.95229909 17.7275038  
## yp[2]       19.30212942 0.0049097757 0.60819513 18.10033198 18.9068894  
## yp[3]       20.05903305 0.0048553720 0.61336573 18.84248048 19.6596314  
## yp[4]       21.35042749 0.0048358749 0.60898330 20.13797460 20.9526223  
## yp[5]       22.79779009 0.0049104084 0.61973592 21.58876713 22.3969766  
## yp[6]       23.86972628 0.0048911105 0.61421029 22.66684387 23.4699709  
## yp[7]       24.31034473 0.0048371705 0.61547047 23.07575686 23.9127524  
## yp[8]       25.35311483 0.0048987297 0.60666761 24.15172968 24.9564350  
## yp[9]       26.74900817 0.0048567951 0.60847954 25.56273823 26.3438555  
## yp[10]      28.11721770 0.0048234575 0.61316173 26.90434121 27.7136833  
## yp[11]      30.59625957 0.0048624912 0.61383000 29.39161709 30.1952590  
## yp[12]      31.82094184 0.0050411449 0.61438899 30.62014768 31.4176258  
## yp[13]      33.23469338 0.0047677236 0.59857110 32.05896564 32.8452393  
## yp[14]      35.39748796 0.0047543471 0.60281682 34.19957785 34.9981804  
## yp[15]      37.38496706 0.0047467199 0.60078544 36.20253121 36.9964036  
## yp[16]      39.87441735 0.0047951665 0.60178818 38.67497383 39.4789639  
## yp[17]      40.69372847 0.0047471922 0.60183713 39.52245584 40.2903429  
## yp[18]      43.35111178 0.0047706287 0.60172409 42.16359628 42.9561560  
## yp[19]      45.46391858 0.0047760680 0.60385322 44.26396376 45.0708258  
## yp[20]      47.42557507 0.0047541036 0.59981509 46.23608877 47.0334206  
## yp[21]      49.04250502 0.0047641284 0.60581695 47.85708942 48.6416057  
## yp[22]      49.91874568 0.0047433300 0.59807750 48.73540526 49.5265205  
## yp[23]      51.51180629 0.0048123656 0.60265924 50.31218998 51.1122390  
## yp[24]      53.68292121 0.0048122746 0.60473827 52.50745683 53.2839020  
## yp[25]      55.18716969 0.0048483433 0.60474128 53.99485364 54.7916975
```

```

## yp[26]      57.27571459 0.0047893353 0.59716448 56.09347638 56.8805637
## yp[27]      57.06822367 0.0056716660 0.65860676 55.77176028 56.6370816
## yp[28]      56.21645705 0.0054314956 0.65463710 54.92547858 55.7851607
## yp[29]      56.03070481 0.0055162111 0.65484564 54.75800060 55.5988801
## yp[30]      58.71149236 0.0048069844 0.59938084 57.52566087 58.3200921
## yp[31]      59.77598407 0.0047673750 0.60408200 58.57930368 59.3787206
## yp[32]      60.00039941 0.0054465295 0.65967986 58.72128117 59.5627692
## yp[33]      61.72056246 0.0048173703 0.60232543 60.52104105 61.3221479
## yp[34]      64.08814705 0.0048399022 0.60676081 62.88786187 63.6829483
## yp[35]      66.58040721 0.0048827840 0.60808047 65.38565441 66.1751511
## yp[36]      68.41848709 0.0072580722 0.87948061 66.67048096 67.8390617
## yp[37]      70.25622793 0.0088588476 1.09951990 68.05733098 69.5370085
## yp[38]      72.09121629 0.0103869756 1.29940467 69.50369229 71.2240541
## yp[39]      73.93297895 0.0118458031 1.48922250 70.99138820 72.9448807
## yp[40]      75.76658929 0.0132599832 1.66297520 72.46032275 74.6764997
## yp[41]      77.60151551 0.0146414952 1.83775364 73.92793750 76.4004250
## yp[42]      79.43936906 0.0160693443 2.00595282 75.45544140 78.1166884
## yp[43]      81.26908753 0.0173826595 2.16550173 76.99914143 79.8365882
## yp[44]      83.10220125 0.0186375349 2.31878963 78.46145109 81.5747222
## yp[45]      84.93427810 0.0199810013 2.47741333 80.02887485 83.3268094
## yp[46]      86.77044327 0.0213500682 2.63949229 81.49528450 85.0405587
## lp__        -4.85278687 0.0328716717 2.33692378 -10.19871169 -6.1991497
##
##              50%      75%      97.5%  n_eff  Rhat
## a              0.79910138 1.1198231 1.7254914 10204.84 0.9998730
## b              0.95439145 1.2599720 2.0926962 10621.40 0.9999137
## beta[1]        1.83581437 1.9273682 2.1036767 13137.87 0.9998292
## beta[2]        0.04182023 0.2331918 0.6549599 11435.57 0.9999255
## beta[3]        1.19088963 1.3062171 1.5298964 14126.04 1.0001478
## sigma_beta[1]  0.66379224 1.1433113 2.2867487 13372.23 1.0000954
## sigma_beta[2]  0.66894970 1.1526202 2.2640154 15492.23 1.0000288
## sigma_beta[3]  0.67773696 1.1369330 2.2297072 14913.61 1.0002350
## sigma_e        0.57453332 0.6300006 0.7609429 11708.17 1.0001235
## yp[1]          18.11558687 18.4954456 19.2771874 16071.50 0.9999885
## yp[2]          19.30417439 19.6996272 20.5090231 15344.85 0.9998916
## yp[3]          20.06307831 20.4561258 21.2534985 15958.57 0.9998867
## yp[4]          21.35566236 21.7485015 22.5418976 15858.45 0.9999893
## yp[5]          22.79850587 23.2091286 24.0168623 15928.62 1.0001105
## yp[6]          23.86475723 24.2679628 25.1038018 15769.55 0.9998712
## yp[7]          24.30847634 24.7223090 25.5095061 16189.43 0.9999814
## yp[8]          25.34744219 25.7497983 26.5462392 15336.80 1.0001125
## yp[9]          26.74544070 27.1461730 27.9445640 15696.12 0.9999004
## yp[10]         28.11637955 28.5213138 29.3411430 16159.70 0.9999369
## yp[11]         30.59543060 30.9964077 31.8135701 15935.97 0.9999706
## yp[12]         31.82037260 32.2225885 33.0466817 14853.49 0.9999039
## yp[13]         33.23493533 33.6338460 34.4080963 15761.93 0.9999675

```

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## yp[14]      35.40158532 35.7932616 36.5800689 16076.41 1.0001021
## yp[15]      37.38726713 37.7711758 38.5790192 16019.60 1.0001647
## yp[16]      39.87836553 40.2673271 41.0476452 15749.98 0.9998232
## yp[17]      40.69214447 41.0863675 41.8954520 16072.53 1.0003743
## yp[18]      43.35751918 43.7466105 44.5347408 15909.03 0.9998675
## yp[19]      45.46438334 45.8604854 46.6533463 15985.34 1.0000937
## yp[20]      47.42257876 47.8197332 48.6206788 15918.33 0.9999544
## yp[21]      49.04534698 49.4428883 50.2260802 16170.22 1.0001644
## yp[22]      49.91293616 50.3098116 51.1235942 15898.21 0.9999767
## yp[23]      51.51274995 51.9182868 52.6723057 15682.90 1.0005622
## yp[24]      53.68404759 54.0752541 54.8763274 15791.89 0.9998654
## yp[25]      55.18935031 55.5808258 56.3799040 15557.95 0.9998773
## yp[26]      57.28583155 57.6709572 58.4292266 15546.67 0.9997841
## yp[27]      57.06512041 57.4974471 58.3973696 13484.38 0.9999800
## yp[28]      56.20764771 56.6410888 57.5193737 14526.55 0.9998619
## yp[29]      56.01960392 56.4599483 57.3450373 14092.76 1.0000709
## yp[30]      58.71122378 59.1082876 59.8865452 15547.49 0.9998584
## yp[31]      59.77959860 60.1732885 60.9623222 16055.84 0.9999808
## yp[32]      59.99264820 60.4330831 61.3052954 14669.89 0.9998162
## yp[33]      61.72380949 62.1182517 62.8937904 15633.00 1.0000891
## yp[34]      64.08827168 64.4917921 65.2785608 15716.72 0.9999648
## yp[35]      66.57926921 66.9847232 67.7796439 15509.12 1.0000249
## yp[36]      68.41883434 69.0061573 70.1665207 14682.84 0.9999366
## yp[37]      70.25375350 70.9806301 72.4152642 15404.65 0.9999052
## yp[38]      72.10373297 72.9481312 74.6189992 15649.87 0.9999593
## yp[39]      73.93351955 74.9200284 76.8814832 15804.84 0.9999890
## yp[40]      75.77634654 76.8733182 79.0091425 15728.44 0.9999684
## yp[41]      77.61256376 78.8233172 81.2409259 15754.47 0.9999546
## yp[42]      79.45897948 80.7710600 83.3829150 15582.79 0.9999035
## yp[43]      81.29718701 82.6944216 85.5041576 15519.75 0.9999136
## yp[44]      83.12446965 84.6448899 87.6900344 15479.10 0.9999719
## yp[45]      84.95180361 86.5828698 89.7960897 15373.14 1.0000185
## yp[46]      86.78803844 88.5146838 91.9165145 15284.18 1.0000072
## lp__        -4.51118804 -3.1351791 -1.3395867 5054.12 1.0003842
##
## $c_summary
## , , chains = chain:1
##
##                stats
## parameter      mean      sd      2.5%      25%      50%
## a              0.76301565 0.53080538 -0.42555849 0.4636588 0.8034816
## b              1.03426931 0.42713406 0.45047957 0.7271913 0.9507955
## beta[1]        1.83531202 0.13494944 1.57248472 1.7469393 1.8328005
## beta[2]        0.04300541 0.28449979 -0.49780031 -0.1512628 0.0347877
## beta[3]        1.19274121 0.17170023 0.84682853 1.0800190 1.1960953

```

##	sigma_beta[1]	0.78489651	0.62116674	0.01921457	0.2851311	0.6526192
##	sigma_beta[2]	0.80163555	0.61641029	0.04037834	0.3158001	0.6608043
##	sigma_beta[3]	0.78112225	0.59896379	0.03151658	0.3177201	0.6567423
##	sigma_e	0.58091563	0.07910206	0.44918828	0.5236748	0.5738646
##	yp[1]	18.11472953	0.60086601	16.94588081	17.7124527	18.1133513
##	yp[2]	19.29582708	0.60506625	18.08549858	18.9021429	19.2974549
##	yp[3]	20.06718633	0.61983626	18.82065692	19.6636130	20.0789564
##	yp[4]	21.34162456	0.59774646	20.13592028	20.9527529	21.3458407
##	yp[5]	22.78652475	0.61732651	21.56920113	22.3870747	22.7986720
##	yp[6]	23.86169851	0.61210192	22.66046674	23.4658658	23.8542536
##	yp[7]	24.30666927	0.62392700	23.04418549	23.8964677	24.3081051
##	yp[8]	25.36420411	0.60493731	24.17058398	24.9685351	25.3572652
##	yp[9]	26.75028318	0.60725070	25.57355106	26.3469147	26.7496378
##	yp[10]	28.12337964	0.61568225	26.89638117	27.7233192	28.1256999
##	yp[11]	30.60531536	0.61580289	29.39800646	30.2066364	30.6033687
##	yp[12]	31.81975679	0.60682619	30.63490434	31.4218642	31.8039816
##	yp[13]	33.24063865	0.59905730	32.04955632	32.8526925	33.2388509
##	yp[14]	35.40958343	0.60469007	34.18693711	35.0161639	35.4300297
##	yp[15]	37.37925720	0.59357226	36.20710853	36.9977972	37.3722806
##	yp[16]	39.87224858	0.59887385	38.66397929	39.4927022	39.8738580
##	yp[17]	40.67919660	0.59600250	39.54178759	40.2708222	40.6770748
##	yp[18]	43.34497304	0.59401660	42.18354007	42.9530816	43.3559125
##	yp[19]	45.46068071	0.59792706	44.26663974	45.0699501	45.4572448
##	yp[20]	47.43061339	0.58579472	46.26198534	47.0421337	47.4260226
##	yp[21]	49.04165855	0.60161318	47.84323119	48.6524572	49.0478616
##	yp[22]	49.90600386	0.60257969	48.73897760	49.5075730	49.8970222
##	yp[23]	51.50747742	0.60681857	50.32507264	51.1101180	51.5051111
##	yp[24]	53.69319223	0.60594415	52.52993934	53.3061127	53.6848682
##	yp[25]	55.17983316	0.58913084	54.01720564	54.8016991	55.1714916
##	yp[26]	57.27618138	0.59663240	56.13359788	56.8845071	57.2746538
##	yp[27]	57.05357853	0.64773048	55.77175152	56.6246167	57.0403223
##	yp[28]	56.21099283	0.64637691	54.92097352	55.7843781	56.1976332
##	yp[29]	56.02009030	0.64861390	54.76897557	55.5959422	56.0014570
##	yp[30]	58.71347560	0.59547916	57.52862760	58.3336149	58.7185633
##	yp[31]	59.78599118	0.60387403	58.57634967	59.3908916	59.7874660
##	yp[32]	60.00469412	0.64695548	58.80292636	59.5741096	59.9901961
##	yp[33]	61.72138766	0.60060551	60.53958010	61.3234442	61.7249547
##	yp[34]	64.10239186	0.60208308	62.88432329	63.7041858	64.1059541
##	yp[35]	66.56502739	0.60643874	65.39586333	66.1529200	66.5636386
##	yp[36]	68.41450114	0.87588551	66.67184364	67.8256104	68.4344948
##	yp[37]	70.27632077	1.09995400	68.06340595	69.5362098	70.2733520
##	yp[38]	72.10380861	1.29491888	69.52772326	71.2545317	72.1206502
##	yp[39]	73.95246120	1.47313722	71.06634408	72.9554440	73.9558900
##	yp[40]	75.78875370	1.64208683	72.59921754	74.6847250	75.7826098
##	yp[41]	77.62958632	1.81742540	74.03470527	76.4025883	77.6686479

```

##   yp[42]      79.46976927  1.96960489  75.73568959  78.1638242  79.4777374
##   yp[43]      81.28848009  2.13394694  77.18941424  79.8410607  81.3369002
##   yp[44]      83.13036857  2.28707245  78.66564953  81.5843968  83.1487200
##   yp[45]      84.97313265  2.46302499  80.12833182  83.3564505  84.9705491
##   yp[46]      86.80694329  2.62499965  81.69716942  85.0537263  86.7741037
##   lp__        -4.80447932  2.32023990 -9.94469963 -6.1403294 -4.4604387
##
##               stats
## parameter          75%          97.5%
##   a                1.1051298  1.7225295
##   b                1.2393817  2.1040397
##   beta[1]          1.9243330  2.1006848
##   beta[2]          0.2271817  0.6254201
##   beta[3]          1.3078328  1.5295745
##   sigma_beta[1]    1.1333080  2.2984143
##   sigma_beta[2]    1.1522734  2.3203671
##   sigma_beta[3]    1.1140197  2.2212222
##   sigma_e          0.6289172  0.7537902
##   yp[1]            18.5078129  19.3374629
##   yp[2]            19.6997766  20.4696394
##   yp[3]            20.4634131  21.2607229
##   yp[4]            21.7293310  22.5181364
##   yp[5]            23.1994412  24.0097210
##   yp[6]            24.2757941  25.0762409
##   yp[7]            24.7254174  25.5153854
##   yp[8]            25.7464346  26.5799987
##   yp[9]            27.1584136  27.9210648
##   yp[10]           28.5390311  29.3140948
##   yp[11]           31.0059788  31.8706383
##   yp[12]           32.2203937  33.0849731
##   yp[13]           33.6385056  34.4398517
##   yp[14]           35.8073670  36.5638102
##   yp[15]           37.7646535  38.5749960
##   yp[16]           40.2735610  41.0283754
##   yp[17]           41.0579883  41.8897348
##   yp[18]           43.7293198  44.5234522
##   yp[19]           45.8516305  46.6448422
##   yp[20]           47.8237019  48.6004664
##   yp[21]           49.4374737  50.2013220
##   yp[22]           50.3001263  51.1211677
##   yp[23]           51.9284347  52.6961085
##   yp[24]           54.0816778  54.9092986
##   yp[25]           55.5592713  56.3798896
##   yp[26]           57.6661500  58.4467494
##   yp[27]           57.4856245  58.3673213
##   yp[28]           56.6326260  57.4735478

```

```

## yp[29] 56.4498107 57.3377912
## yp[30] 59.0967904 59.8910759
## yp[31] 60.1854046 60.9651031
## yp[32] 60.4293428 61.2821626
## yp[33] 62.1079681 62.9043882
## yp[34] 64.5000266 65.2633618
## yp[35] 66.9868071 67.7287993
## yp[36] 69.0134648 70.1495300
## yp[37] 71.0079294 72.4173194
## yp[38] 72.9655578 74.5933989
## yp[39] 74.9253658 76.9142427
## yp[40] 76.8934602 79.0372706
## yp[41] 78.8061157 81.2933469
## yp[42] 80.7772364 83.4692719
## yp[43] 82.6819463 85.5314091
## yp[44] 84.6439254 87.6036673
## yp[45] 86.6147229 89.7796088
## yp[46] 88.5415482 92.0181122
## lp__ -3.1182366 -1.2412386
##
## , , chains = chain:2
##
## stats
## parameter mean sd 2.5% 25% 50%
## a 0.75279822 0.54510342 -0.47660306 0.4327810 0.78984072
## b 1.03698110 0.43617523 0.43867098 0.7126657 0.95860966
## beta[1] 1.83660363 0.14000785 1.55674055 1.7447411 1.83782449
## beta[2] 0.04460026 0.29530801 -0.53006030 -0.1485164 0.03728099
## beta[3] 1.18901628 0.17489453 0.84135129 1.0698654 1.18971696
## sigma_beta[1] 0.80984566 0.59900286 0.03564096 0.3305929 0.68812122
## sigma_beta[2] 0.80282962 0.59868978 0.03338321 0.3252682 0.68349640
## sigma_beta[3] 0.80128939 0.59710066 0.03776911 0.3370528 0.67616701
## sigma_e 0.58203973 0.07830184 0.45212351 0.5271307 0.57412895
## yp[1] 18.10057093 0.57390016 16.95121874 17.7347772 18.09270712
## yp[2] 19.30001851 0.60812288 18.09946227 18.9140099 19.30868774
## yp[3] 20.05296854 0.60921276 18.86482713 19.6500744 20.04101514
## yp[4] 21.33605914 0.61583895 20.12598053 20.9207031 21.33022616
## yp[5] 22.79941892 0.62371017 21.56770798 22.3971244 22.80306492
## yp[6] 23.87278732 0.62020657 22.66273604 23.4655081 23.86704538
## yp[7] 24.30067840 0.62316412 23.02951410 23.8964124 24.29681345
## yp[8] 25.35904297 0.60879695 24.14624694 24.9689966 25.34384275
## yp[9] 26.73543763 0.61513655 25.53314692 26.3169726 26.73107879
## yp[10] 28.10453953 0.60930675 26.88531890 27.7072879 28.10215169
## yp[11] 30.58755106 0.60637331 29.41029583 30.1953424 30.58714450
## yp[12] 31.82081941 0.61361122 30.62913802 31.4148866 31.83220800

```

```

## yp[13] 33.22531726 0.59974114 32.04550944 32.8275244 33.22519141
## yp[14] 35.38766177 0.60968571 34.15115485 34.9867237 35.38890431
## yp[15] 37.39651549 0.59383282 36.20630656 37.0155421 37.39555563
## yp[16] 39.88391022 0.59820484 38.70962534 39.4806055 39.88319232
## yp[17] 40.69745684 0.60067024 39.50662778 40.2909825 40.69494409
## yp[18] 43.35123250 0.60222616 42.15691316 42.9619880 43.34852652
## yp[19] 45.46494374 0.60267894 44.32490354 45.0741274 45.45930631
## yp[20] 47.43482085 0.59269971 46.27919686 47.0418412 47.42597208
## yp[21] 49.04566502 0.61781607 47.82864703 48.6424701 49.05197692
## yp[22] 49.91988758 0.60046114 48.71975775 49.5380898 49.91271012
## yp[23] 51.50454617 0.59970473 50.32550002 51.0919908 51.50461623
## yp[24] 53.68473439 0.60430505 52.50707616 53.2731643 53.68998025
## yp[25] 55.19051895 0.60557012 54.01951913 54.7859156 55.19519628
## yp[26] 57.27495128 0.60020608 56.05718701 56.8744266 57.28976566
## yp[27] 57.07962354 0.65632526 55.79992172 56.6476018 57.07651234
## yp[28] 56.20665806 0.65710406 54.91493336 55.7744353 56.19648829
## yp[29] 56.02935867 0.66347024 54.74041751 55.5857424 56.02472248
## yp[30] 58.70808866 0.60233081 57.51070714 58.3135662 58.70986083
## yp[31] 59.78231281 0.60294594 58.59070272 59.3948839 59.78741835
## yp[32] 60.00077017 0.65806115 58.69960707 59.5731207 60.00131042
## yp[33] 61.70261550 0.59065607 60.51383614 61.3217898 61.70157900
## yp[34] 64.09416741 0.61136417 62.92507508 63.6835911 64.08910787
## yp[35] 66.59491036 0.59938483 65.42826217 66.1953661 66.59929283
## yp[36] 68.42300622 0.86967878 66.73757325 67.8626158 68.41230147
## yp[37] 70.25437819 1.09675806 68.08436658 69.5526981 70.23752321
## yp[38] 72.10468521 1.30099174 69.50913303 71.2209148 72.09496143
## yp[39] 73.94910092 1.49657348 71.04480445 72.9434996 73.96023943
## yp[40] 75.77972521 1.67934071 72.49764335 74.6839904 75.81323760
## yp[41] 77.61017475 1.86991224 73.80383745 76.4257685 77.61528191
## yp[42] 79.44078208 2.05559074 75.24272507 78.0822643 79.46341004
## yp[43] 81.27291628 2.22596298 76.88577565 79.8191903 81.29973712
## yp[44] 83.11449506 2.39136178 78.32105039 81.5497302 83.13943909
## yp[45] 84.95339440 2.55198693 79.85028109 83.2838684 84.98800557
## yp[46] 86.79838688 2.71554611 81.39452981 85.0246013 86.85059160
## lp__ -4.76372181 2.28401230 -9.92364046 -6.1134727 -4.41344866
##
## stats
## parameter 75% 97.5%
## a 1.1224579 1.7236074
## b 1.2695003 2.1013733
## beta[1] 1.9309185 2.1120719
## beta[2] 0.2315834 0.6549992
## beta[3] 1.3055655 1.5303322
## sigma_beta[1] 1.1582822 2.2462199
## sigma_beta[2] 1.1554686 2.2359877
## sigma_beta[3] 1.1402957 2.2242533

```

```

## sigma_e      0.6285099  0.7562304
## yp[1]        18.4785195 19.2115569
## yp[2]        19.6946774 20.4770617
## yp[3]        20.4401297 21.2317386
## yp[4]        21.7353837 22.5436006
## yp[5]        23.2050343 24.0420551
## yp[6]        24.2732810 25.1058195
## yp[7]        24.7190235 25.5124293
## yp[8]        25.7581756 26.5798729
## yp[9]        27.1453875 27.9428371
## yp[10]       28.5068986 29.3213504
## yp[11]       30.9833534 31.7993906
## yp[12]       32.2234714 33.0409864
## yp[13]       33.6282955 34.3995060
## yp[14]       35.7791665 36.5454029
## yp[15]       37.7782948 38.5756220
## yp[16]       40.2735909 41.0588420
## yp[17]       41.1013111 41.8783948
## yp[18]       43.7483375 44.5161551
## yp[19]       45.8542167 46.6974168
## yp[20]       47.8199214 48.6146492
## yp[21]       49.4473055 50.2567118
## yp[22]       50.3073786 51.1490070
## yp[23]       51.9199743 52.6611660
## yp[24]       54.0806058 54.8739485
## yp[25]       55.6045850 56.3572678
## yp[26]       57.6833275 58.3846889
## yp[27]       57.5005323 58.3824191
## yp[28]       56.6273214 57.5515446
## yp[29]       56.4555784 57.3324771
## yp[30]       59.1028139 59.8884878
## yp[31]       60.1730142 60.9756078
## yp[32]       60.4124193 61.2896652
## yp[33]       62.0959618 62.8362086
## yp[34]       64.5009264 65.3132013
## yp[35]       66.9882946 67.7878748
## yp[36]       69.0048877 70.1859186
## yp[37]       70.9628728 72.4454807
## yp[38]       72.9560160 74.6503571
## yp[39]       74.9458079 76.8892157
## yp[40]       76.8824939 79.0424257
## yp[41]       78.8881120 81.2281006
## yp[42]       80.8099986 83.4640193
## yp[43]       82.7445922 85.5989325
## yp[44]       84.7042046 87.7445734

```

```

##   yp[45]      86.6547359 89.9979792
##   yp[46]      88.5976668 92.1246922
##   lp__        -3.0566914 -1.3908635
##
## , , chains = chain:3
##
##           stats
## parameter      mean      sd      2.5%      25%      50%
## a              0.76663002 0.5412796 -0.40515662 0.4385652 0.8116530
## b              1.03763573 0.4200994 0.44362483 0.7370682 0.9529792
## beta[1]        1.83485958 0.1381182 1.55719746 1.7463016 1.8362289
## beta[2]        0.04916291 0.3009399 -0.54393667 -0.1489435 0.0506301
## beta[3]        1.19257400 0.1758214 0.84127243 1.0764438 1.1947806
## sigma_beta[1] 0.78824171 0.6144413 0.02094041 0.3018050 0.6557160
## sigma_beta[2] 0.80338975 0.6127142 0.03390464 0.3174974 0.6608804
## sigma_beta[3] 0.80290136 0.6108481 0.02738237 0.3157432 0.6865543
## sigma_e        0.58473727 0.0792792 0.45310291 0.5279332 0.5772032
## yp[1]          18.11417764 0.5862056 16.96161911 17.7267433 18.1210164
## yp[2]          19.30962965 0.6108184 18.13993122 18.9074258 19.3037741
## yp[3]          20.05439600 0.6243388 18.81133847 19.6474871 20.0580721
## yp[4]          21.36084986 0.6169260 20.10877538 20.9632988 21.3689132
## yp[5]          22.80425451 0.6205703 21.59473956 22.4015736 22.7972348
## yp[6]          23.87302257 0.6032893 22.70501931 23.4720001 23.8732451
## yp[7]          24.32589950 0.6091796 23.11158063 23.9384013 24.3206963
## yp[8]          25.33908434 0.6055938 24.16255066 24.9377367 25.3436425
## yp[9]          26.75393874 0.6101759 25.55316959 26.3509643 26.7466582
## yp[10]         28.12391166 0.6145183 26.90381490 27.7287559 28.1208337
## yp[11]         30.60550630 0.6230474 29.35176093 30.1975131 30.6213335
## yp[12]         31.82361348 0.6160346 30.61226036 31.4211624 31.8276551
## yp[13]         33.22850546 0.6023752 32.06208548 32.8312545 33.2303701
## yp[14]         35.40817288 0.6013212 34.19953825 35.0030147 35.4036906
## yp[15]         37.36802879 0.6094927 36.19658505 36.9706589 37.3723047
## yp[16]         39.87437436 0.6041134 38.69753173 39.4747471 39.8730876
## yp[17]         40.71682006 0.6162753 39.52745741 40.3061178 40.7114362
## yp[18]         43.35039166 0.6069821 42.16347230 42.9490945 43.3585200
## yp[19]         45.46717320 0.6101323 44.23468271 45.0649597 45.4794443
## yp[20]         47.41607596 0.6159619 46.20158037 47.0088264 47.4203651
## yp[21]         49.04416229 0.6036761 47.86863248 48.6310905 49.0411048
## yp[22]         49.93693621 0.5979152 48.77043452 49.5432906 49.9323116
## yp[23]         51.52677350 0.5997215 50.29842835 51.1335087 51.5295939
## yp[24]         53.68330676 0.6067318 52.50760375 53.2808237 53.6948093
## yp[25]         55.19362913 0.6081858 53.98137162 54.7923055 55.2015280
## yp[26]         57.27895707 0.6038664 56.07767496 56.8710340 57.2951485
## yp[27]         57.05928955 0.6624974 55.75863847 56.6270662 57.0522588
## yp[28]         56.22523244 0.6512552 54.94487021 55.7930207 56.2177602

```

```

##   yp[29]      56.03589865  0.6620932  54.72634405  55.6051447  56.0273071
##   yp[30]      58.71712383  0.6077305  57.55624552  58.3111059  58.7109802
##   yp[31]      59.76575769  0.6119765  58.54520593  59.3661904  59.7718305
##   yp[32]      59.99662398  0.6745225  58.68418574  59.5443571  59.9977170
##   yp[33]      61.73007616  0.6067415  60.51864358  61.3249623  61.7388588
##   yp[34]      64.07657617  0.6097925  62.85947782  63.6688219  64.0769167
##   yp[35]      66.58296924  0.6144908  65.37169583  66.1823377  66.5779773
##   yp[36]      68.41970824  0.8791431  66.66126329  67.8330365  68.4131562
##   yp[37]      70.24818029  1.0990468  68.08756390  69.5367387  70.2507637
##   yp[38]      72.09065518  1.2928385  69.56435694  71.2106032  72.1013800
##   yp[39]      73.94309110  1.4918738  70.95315905  72.9494269  73.9350474
##   yp[40]      75.77282624  1.6641430  72.44485361  74.6553136  75.8054223
##   yp[41]      77.60371086  1.8392175  74.00351673  76.3660624  77.6115552
##   yp[42]      79.44174410  2.0090389  75.49350151  78.0909107  79.4513205
##   yp[43]      81.27400389  2.1620970  77.03313263  79.8180607  81.2998810
##   yp[44]      83.08393081  2.3074071  78.55768541  81.5659253  83.1020062
##   yp[45]      84.90412764  2.4601132  80.13006919  83.2993593  84.9050590
##   yp[46]      86.73993687  2.6211134  81.62517617  85.0150352  86.7284419
##   lp__        -4.94711992  2.3791897 -10.45060660 -6.2830180 -4.5732433
##
##               stats
## parameter      75%      97.5%
##   a             1.1305519  1.7430970
##   b             1.2538917  2.0916849
##   beta[1]       1.9293908  2.0986515
##   beta[2]       0.2449699  0.6629046
##   beta[3]       1.3136300  1.5306086
##   sigma_beta[1] 1.1256056  2.3158830
##   sigma_beta[2] 1.1684580  2.2523672
##   sigma_beta[3] 1.1575050  2.2923459
##   sigma_e       0.6322494  0.7582254
##   yp[1]        18.4917954  19.2675457
##   yp[2]        19.7047031  20.5407863
##   yp[3]        20.4629202  21.2989266
##   yp[4]        21.7608699  22.5885868
##   yp[5]        23.2203868  24.0030975
##   yp[6]        24.2628117  25.0710889
##   yp[7]        24.7220517  25.5364189
##   yp[8]        25.7267649  26.5277760
##   yp[9]        27.1514652  27.9433954
##   yp[10]       28.5307763  29.3550602
##   yp[11]       31.0137973  31.8246331
##   yp[12]       32.2330414  33.0341973
##   yp[13]       33.6269022  34.4099900
##   yp[14]       35.8111527  36.5949806
##   yp[15]       37.7534280  38.5833346

```

```

## yp[16] 40.2604373 41.0554642
## yp[17] 41.1249505 41.9421392
## yp[18] 43.7482960 44.5781103
## yp[19] 45.8816383 46.6147626
## yp[20] 47.8243787 48.6176303
## yp[21] 49.4431675 50.2404792
## yp[22] 50.3308182 51.1183620
## yp[23] 51.9174469 52.7129930
## yp[24] 54.0704916 54.8766152
## yp[25] 55.5825620 56.3779498
## yp[26] 57.6860888 58.4633407
## yp[27] 57.4947769 58.3696006
## yp[28] 56.6411505 57.4948777
## yp[29] 56.4794086 57.3970752
## yp[30] 59.1285371 59.9225290
## yp[31] 60.1703382 60.9533211
## yp[32] 60.4411363 61.3341911
## yp[33] 62.1200619 62.9366396
## yp[34] 64.4887916 65.2546290
## yp[35] 66.9788391 67.8163483
## yp[36] 69.0053726 70.2033723
## yp[37] 70.9758467 72.3756248
## yp[38] 72.9551018 74.6122944
## yp[39] 74.9332937 76.8854384
## yp[40] 76.8967672 79.0371291
## yp[41] 78.8590623 81.2371966
## yp[42] 80.7987278 83.4470731
## yp[43] 82.7356294 85.5174298
## yp[44] 84.6376463 87.7426893
## yp[45] 86.5270540 89.8239524
## yp[46] 88.4534556 91.8863573
## lp__ -3.2177588 -1.3575406
##
## , , chains = chain:4
##
## stats
## parameter mean sd 2.5% 25% 50%
## a 0.75098609 0.54554394 -0.45672018 0.4311530 0.79380976
## b 1.03735038 0.43056609 0.43833018 0.7199797 0.95388066
## beta[1] 1.83381344 0.13714198 1.56651555 1.7437088 1.83625378
## beta[2] 0.05258037 0.29489165 -0.51216662 -0.1437956 0.04603255
## beta[3] 1.18461910 0.17065536 0.84000195 1.0783122 1.18475058
## sigma_beta[1] 0.79094148 0.61291657 0.02830166 0.3001033 0.65652781
## sigma_beta[2] 0.78347330 0.60542940 0.02082040 0.3020707 0.66550858
## sigma_beta[3] 0.79906123 0.59055399 0.03040868 0.3350859 0.68406833

```

##	sigma_e	0.58207335	0.08129228	0.44614490	0.5262678	0.57369636
##	yp[1]	18.12119891	0.58727579	16.94855047	17.7393727	18.12553654
##	yp[2]	19.30304246	0.60890372	18.09933136	18.9055954	19.30865130
##	yp[3]	20.06158134	0.59990411	18.86583555	19.6747395	20.07281685
##	yp[4]	21.36317638	0.60498719	20.15874717	20.9767589	21.37531212
##	yp[5]	22.80096218	0.61740039	21.59581626	22.4033381	22.79655907
##	yp[6]	23.87139672	0.62123244	22.64975788	23.4749311	23.86809167
##	yp[7]	24.30813173	0.60533269	23.11692290	23.9154236	24.30901954
##	yp[8]	25.35012791	0.60726256	24.12661432	24.9539543	25.34481433
##	yp[9]	26.75637313	0.60128334	25.59276088	26.3623245	26.75260561
##	yp[10]	28.11703998	0.61315218	26.92842593	27.6999982	28.11777801
##	yp[11]	30.58666556	0.60992328	29.42124298	30.1838315	30.57963739
##	yp[12]	31.81957768	0.62121866	30.60157919	31.4148326	31.82206973
##	yp[13]	33.24431215	0.59308439	32.07280515	32.8567438	33.24668513
##	yp[14]	35.38453378	0.59526982	34.23154734	34.9850294	35.38704556
##	yp[15]	37.39606675	0.60582207	36.20282133	37.0000017	37.40464609
##	yp[16]	39.86713623	0.60602634	38.63564137	39.4715462	39.88397147
##	yp[17]	40.68144038	0.59361172	39.51623325	40.2947712	40.67575449
##	yp[18]	43.35784994	0.60375172	42.16300145	42.9594637	43.36506012
##	yp[19]	45.46287667	0.60481817	44.24559301	45.0725969	45.45829823
##	yp[20]	47.42079009	0.60440133	46.21585854	47.0299448	47.41744130
##	yp[21]	49.03853422	0.60020236	47.89084004	48.6389222	49.03205071
##	yp[22]	49.91215508	0.59106554	48.72062163	49.5248934	49.91086762
##	yp[23]	51.50842807	0.60433251	50.27680190	51.1172854	51.50958665
##	yp[24]	53.67045146	0.60196920	52.49634693	53.2656086	53.66557297
##	yp[25]	55.18469752	0.61589702	53.95271726	54.7879047	55.18917648
##	yp[26]	57.27276863	0.58804509	56.10389051	56.8906615	57.28572902
##	yp[27]	57.08040307	0.66752051	55.77009154	56.6526358	57.08435160
##	yp[28]	56.22294486	0.66374181	54.90729725	55.7930723	56.21205368
##	yp[29]	56.03747163	0.64510890	54.77714910	55.6118108	56.01949550
##	yp[30]	58.70728136	0.59203030	57.50760539	58.3177740	58.70485989
##	yp[31]	59.76987460	0.59743531	58.59458306	59.3695522	59.77055624
##	yp[32]	59.99950938	0.65911031	58.70401110	59.5737079	59.98650336
##	yp[33]	61.72817050	0.61094009	60.51342739	61.3201110	61.72904980
##	yp[34]	64.07945275	0.60361003	62.89932442	63.6825081	64.07565862
##	yp[35]	66.57872184	0.61175105	65.34870829	66.1730495	66.57364214
##	yp[36]	68.41673274	0.89335024	66.61836137	67.8368670	68.42288225
##	yp[37]	70.24603248	1.10246363	68.02378494	69.5212496	70.25884760
##	yp[38]	72.06571617	1.30891560	69.39990951	71.2151863	72.08411857
##	yp[39]	73.88726259	1.49479529	70.93924216	72.9187175	73.87647544
##	yp[40]	75.72505202	1.66600893	72.33245858	74.6825614	75.72459123
##	yp[41]	77.56259011	1.82405477	73.87516798	76.4066129	77.56929193
##	yp[42]	79.40518079	1.98878611	75.44042543	78.1402969	79.43846921
##	yp[43]	81.24094987	2.13929840	76.91126883	79.8709737	81.26756904
##	yp[44]	83.08001056	2.28822304	78.37640283	81.6268503	83.11581093

```

##   yp[45]      84.90645770  2.43311864  79.86655739  83.3822424  84.93216459
##   yp[46]      86.73650605  2.59492944  81.25948821  85.0634492  86.76438096
##   lp__        -4.89582642  2.35948217 -10.31737493 -6.2652662  -4.56883436
##
##               stats
## parameter          75%          97.5%
##   a                1.1216038  1.7006472
##   b                1.2707606  2.0741851
##   beta[1]          1.9244611  2.1032557
##   beta[2]          0.2350652  0.6658572
##   beta[3]          1.2954936  1.5273020
##   sigma_beta[1]    1.1620819  2.2091987
##   sigma_beta[2]    1.1220871  2.2279455
##   sigma_beta[3]    1.1392566  2.1665266
##   sigma_e          0.6300375  0.7656389
##   yp[1]            18.5050478  19.2936159
##   yp[2]            19.6967784  20.5278520
##   yp[3]            20.4527797  21.2280461
##   yp[4]            21.7669877  22.5300722
##   yp[5]            23.2138420  24.0080958
##   yp[6]            24.2606112  25.1666089
##   yp[7]            24.7218622  25.4799896
##   yp[8]            25.7685145  26.5044328
##   yp[9]            27.1326495  27.9596704
##   yp[10]           28.5084930  29.3846861
##   yp[11]           30.9855852  31.7940339
##   yp[12]           32.2131305  33.0585448
##   yp[13]           33.6426278  34.3879064
##   yp[14]           35.7708301  36.6081896
##   yp[15]           37.7929188  38.5819362
##   yp[16]           40.2612188  41.0442040
##   yp[17]           41.0642496  41.8408233
##   yp[18]           43.7609345  44.5319918
##   yp[19]           45.8514214  46.6494195
##   yp[20]           47.8109771  48.6293683
##   yp[21]           49.4409022  50.2084522
##   yp[22]           50.2974520  51.0904282
##   yp[23]           51.9083982  52.6424797
##   yp[24]           54.0684792  54.8360747
##   yp[25]           55.5809467  56.4201341
##   yp[26]           57.6473353  58.4156404
##   yp[27]           57.5079000  58.4374420
##   yp[28]           56.6657655  57.5600339
##   yp[29]           56.4569859  57.3090557
##   yp[30]           59.1051080  59.8443467
##   yp[31]           60.1672460  60.9623718

```

```
## yp[32] 60.4502349 61.2908719
## yp[33] 62.1484739 62.9098122
## yp[34] 64.4826114 65.2758129
## yp[35] 66.9828158 67.7558602
## yp[36] 69.0015054 70.1746821
## yp[37] 70.9805075 72.4065523
## yp[38] 72.9215479 74.5907713
## yp[39] 74.8703104 76.8378556
## yp[40] 76.8228161 78.9315124
## yp[41] 78.7426679 81.1953225
## yp[42] 80.7102591 83.2743694
## yp[43] 82.6363518 85.3748223
## yp[44] 84.6008642 87.5490990
## yp[45] 86.5132409 89.6280219
## yp[46] 88.4582554 91.6786198
## lp__ -3.1712986 -1.3603110
```